USM-TH-89

Some remarks about gauge-invariant Yang-Mills fields

Patricio Gaete *

Departamento de Física, Universidad Técnica F. Santa Maria, Casilla 110-V, Valparaiso, Chile

Abstract

In order to eliminate gauge variant degrees of freedom we study the way to introduce gauge invariant fields in pure non-Abelian Yang-Mills theory. Our approach is based on the use of the gauge-invariant but path-dependent variables formalism. It is shown that for a special class of paths these fields coincide with the usual ones in some definite gauges. The interquark potential is discussed by exploiting the rich structure of the gluonic cloud or dressing around static fermions.

PACS number(s): 12.20.Ds, 11.15.Tk

I. INTRODUCTION

The property of gauge invariance is at the origin of one of the important advances of theoretical physics, that is, with the introduction of non-Abelian gauge theories it was possible the development of quantum chromodynamics (QCD) as the theory of strong interactions, whose properties at short distances can be calculated from perturbation theory. As is well known, for short distances the property of asymptotic freedom is valid, and this explains why perturbation theory can be used at sufficiently large values of the momenta. But within

*E-mail: pgaete@fis.utfsm.cl

this framework, we cannot explain low energy phenomena such as the permanent confinement of quarks and gluons. The analysis of this problem directly from the QCD lagrangian is extremely difficult. The reason is that the infrared divergences and gauge dependence make bound-state equations very hard to approximate. Along the same line, we also recall that the choice of the gauge has a strong influence on the properties of the propagator [1]. An illustrative example on this subject arises when one considers the infrared behavior of the fermion propagator in QCD2. In fact, it has been emphasized that ambiguities appears with the choice of the infrared regularization when a gauge-dependent fermion propagator is treated in the $1/N$ approximation [2]. We also recall that the studies of Caracciolo et al. [3] were crucial in the investigation of the ambiguity in the definition of the gluon propagator. As a way to circumvent these difficulties there has been a recent renewal of interest in formulations of QCD in which gauge-invariant variables are explicitly constructed. By doing so it has been possible to obtain a more deeper and illuminating view on the description of charged particles [4,5]. It should be noted that the picture which emerges from these studies is that quarks are dressed objects, where this dressing is viewed as surrounding the quark with a cloud of gauge fields.

In previous work [6–8] we have investigated the gauge-invariant but path-dependent formalism in Abelian gauge theories, and the intimately related question of gauge fixing. We illustrated how the gauge fixing procedure corresponds, in this formalism, to a path choice. We developed a path-dependent but physical QED where a consistent quantization directly in the path space was carried out. It is worthwhile remarking at this point that the physical electron (dressed) is not the Lagrangian fermion, which is neither gauge invariant nor associated with an electric field. Instead, the physical electron is the Lagrangian fermion accompanied with a non-local cloud of gauge fields. Following this line of argument we reconsidered the calculation of the interaction energy in pure QED and Maxwell-Chern-Simons gauge theory, paying due attention to the structure of the fields that surround the charges. In such a case subtleties related to the problem of exhibiting explicitly the interaction energies were illustrated. Subsequently, we have considered QED2 with associated issues such as

screening and confinement, producing computational rules that have clear as well as simple interpretations [8].

Inspired by these observations, in this Brief Report we further pursue the gauge-invariant but path-dependent variables formalism by studying the extension to non-Abelian gauge fields. In Sec.II the gauge-invariant variables are introduced. The general approach is similar to that of other authors [2,9,10], paying due attention to the question of gauge choice. As an application of the formalism, we consider the problem of exhibiting explicitly the Coulombic interaction between pointlike sources in a non-Abelian pure Yang-Mills theory (gluodynamics) in Sec.III. As already expressed these variables are non-local and require the introduction of strings to carry electric flux.

II. GAUGE-INVARIANT FIELDS

A. The Poincaré gauge

The aim of this section is to discuss the way to introduce gauge-invariant fields in Yang-Mills theory. Towards such an end we will begin by giving a brief account of some of the previous work on Abelian gauge-invariant fields [6] which is relevant to the work at hand. We also recall that our line of thought is to construct variables which are themselves unaltered by a gauge transformation. Accordingly, we consider the gauge-invariant field

$$\mathcal{A}_\mu(x) = A_\mu(x) + \partial_\mu \left(-\int_{C_{\xi x}} dz^\nu A_\nu(z) \right), \tag{1}$$

where the path integral is to be evaluated along some contour $C_{\xi x}$ connecting ξ and x. Here A_μ is the usual electromagnetic potential and, in principle, it is taken in an arbitrary gauge. What we would like to stress is that by choosing a spacelike path from the point ξ^k to x^k, on a fixed time slice, the expression (1) may be rewritten as :

$$\mathcal{A}_0\left(x,\xi\right) = -\int_0^1 d\lambda \left(x-\xi\right)^k F_{0k}\left(\xi + \lambda\left(x-\xi\right)\right), \tag{2}$$

$$\mathcal{A}_i(x,\xi) = -\int_0^1 d\lambda\lambda\,(x-\xi)^k F_{ki}(\xi+\lambda(x-\xi)), \tag{3}$$

where λ ($0 \leq \lambda \leq 1$) is the parameter describing the contour $z^k = \xi^k + \lambda(x-\xi)^k$ with $k = 1,2,3$; and, as usual, $F_{\mu\nu}$ stands for the field strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Thus, as seen from (2) and (3), the potentials $\mathcal{A}_\mu(x,\xi)$ are expressed in a simple way in terms of the gauge field strengths $F_{\mu\nu}$. In passing we note that the field strengths are gauge-invariant observables in the Abelian case, in contrast to the non-Abelian theories where the field strengths are gauge covariant rather than invariant. This raises the question of how to construct gauge-invariant fields in the non-Abelian case. It is the purpose of the present section to provide such construction. Before going into details, we would like to add here that the expressions (2) and (3) coincide with the Poincaré gauge conditions [11] defined by

$$(x-\xi)^i A_i(x) = 0, \tag{4}$$

$$A_0(x) = \int_0^1 d\lambda\,(x-\xi)^i \Pi^i(\lambda x), \tag{5}$$

where Π^i is the conjugate momentum to A^i.

Our immediate undertaking is to extend the above derivation to the non-Abelian case. The first step in this direction is to define the following fields

$$\begin{aligned} \mathcal{A}_0(x,\xi) &= A_0(x) - \partial_0 \int_0^1 d\lambda\,(x-\xi)^k A_k(\xi+\lambda(x-\xi)) + \\ &+ ig\int_0^1 d\lambda\,(x-\xi)^k \left[A_0(\xi+\lambda(x-\xi)), A_k(\xi+\lambda(x-\xi))\right], \end{aligned} \tag{6}$$

$$\begin{aligned} \mathcal{A}_i(x,\xi) &= A_i(x) - \partial_i \int_0^1 d\lambda\,(x-\xi)^k A_k(\xi+\lambda(x-\xi)) + \\ &+ ig\int_0^1 d\lambda\lambda\,(x-\xi)^k \left[A_i(\xi+\lambda(x-\xi)), A_k(\xi+\lambda(x-\xi))\right]. \end{aligned} \tag{7}$$

To transform (7) we use

$$G_{\mu\nu}\left(x\right)=\partial_{\mu}A_{\nu}\left(x\right)-\partial_{\mu}A_{\nu}\left(x\right)-ig\left[A_{\mu}\left(x\right),A_{\nu}\left(x\right)\right], \tag{8}$$

which allows us to rewrite (7) in the form

$$A_{i}\left(x,\xi\right)=A_{i}\left(x\right)-\int\limits_{0}^{1}d\lambda\frac{d}{d\lambda}\left(\lambda A_{i}\left(\xi+\lambda\left(x-\xi\right)\right)\right)+$$
$$-\int\limits_{0}^{1}d\lambda\lambda\left(x-\xi\right)^{k}G_{ik}\left(\xi+\lambda\left(x-\xi\right)\right). \tag{9}$$

The first integral in (9) is found immediately, and it exactly cancels the first term of (9), so that

$$A_{i}\left(x,\xi\right)=\int\limits_{0}^{1}d\lambda\lambda\left(x-\xi\right)^{k}G_{ki}\left(\xi+\lambda\left(x-\xi\right)\right). \tag{10}$$

Proceeding in a similar manner, one gets the following expression for (6),

$$A_{0}\left(x,\xi\right)=-\int\limits_{0}^{1}d\lambda\left(x-\xi\right)^{k}G_{0k}\left(\xi+\lambda\left(x-\xi\right)\right). \tag{11}$$

Except for the substitution of $F_{\mu\nu}$ to $G_{\mu\nu}$ we see that the non-Abelian inversion formula for the fields (6) and (7) coincides with (2) and (3). In fact, such expressions are the non-Abelian inversion formula in the Poincaré gauge by application of the Poincaré lemma to the field strength two-form [12]. However, as outlined above, the fields (10) and (11) are not gauge invariant. Thus, as a possible way to introduce gauge invariant fields, we perform the gauge transformations

$$A_{\mu}\left(x\right)\rightarrow A_{\mu}^{\Lambda}\left(x\right)=\Lambda^{-1}\left(x\right)\left(A_{\mu}\left(x\right)-\frac{i}{g}\partial_{\mu}\right)\Lambda\left(x\right), \tag{12}$$

$$G_{\mu\nu}\left(x\right)\rightarrow G_{\mu\nu}^{\Lambda}\left(x\right)=\Lambda\left(x\right)G_{\mu\nu}\left(x\right)\Lambda^{-1}\left(x\right), \tag{13}$$

with the gauge transformation $\Lambda\left(x\right)=U^{\dagger}\left(x,\xi\right)$. The operator $U(x,\xi)$ is defined by the P-ordered exponential

$$U\left(x,\xi\right)=P\exp\left(-ig\int\limits_{\xi}^{x}dz^{i}A_{i}\left(z\right)\right), \tag{14}$$

with the integration path corresponding to the spacelike straight line which connects ξ to x. Let us also recall here that the P-ordered exponential transforms under a gauge transformation $\Lambda(x)$ as

$$U(x,\xi) \rightarrow U^{\Lambda}(x,\xi) = \Lambda(x)\, U(x,\xi)\, \Lambda^{-1}(\xi). \tag{15}$$

As a consequence $G_{0k}(x,\xi)$ and $G_{ki}(x,\xi)$ take the form

$$G_{0k}(x,\xi) \rightarrow G_{0k}^{U}(x,\xi) \equiv \mathcal{G}_{0k}(x,\xi) = U^{\dagger}(x,\xi)\, G_{ok}(x,\xi)\, U(x,\xi), \tag{16}$$

$$G_{ki}(x,\xi) \rightarrow G_{ki}^{U}(x,\xi) \equiv \mathcal{G}_{ki}(x,\xi) = U^{\dagger}(x,\xi)\, G_{ki}(x,\xi)\, U(x,\xi), \tag{17}$$

and similarly the fields $A_0(x,\xi)$ and $A_i(x,\xi)$ will transform into $\mathcal{A}_0(x,\xi)$ and $\mathcal{A}_i(x,\xi)$ respectively. The point we wish to emphasize, however, is that under the local gauge transformations the fields (16) and (17) transform globally. To see this, let us study how a gauge transformation affects the fields (16) and (17), that is,

$$\mathcal{G}_{0k}(x,\xi) \rightarrow \mathcal{G}_{0k}^{\Lambda}(x,\xi) = \left(U^{\dagger}(x,\xi)\right)^{\Lambda} \mathcal{G}_{0k}^{\Lambda}(x,\xi)\, U^{\Lambda}(x,\xi), \tag{18}$$

$$\mathcal{G}_{ki}(x,\xi) \rightarrow \mathcal{G}_{ki}^{\Lambda}(x,\xi) = \left(U^{\dagger}(x,\xi)\right)^{\Lambda} \mathcal{G}_{ki}^{\Lambda}(x,\xi)\, U^{\Lambda}(x,\xi). \tag{19}$$

Using the expressions (12) and (15) a short calculation yields

$$\mathcal{G}_{0k}^{\Lambda}(x,\xi) = \Lambda(\xi)\, \mathcal{G}_{0k}(x,\xi)\, \Lambda^{-1}(\xi), \tag{20}$$

$$\mathcal{G}_{ki}^{\Lambda}(x,\xi) = \Lambda(\xi)\, \mathcal{G}_{ki}(x,\xi)\, \Lambda^{-1}(\xi). \tag{21}$$

The above result explicitly shows that the fields (16) and (17), under local gauge transformations, transform only globally. This allows us to say that the gauge invariance of (20) and (21) is retrieved when $\Lambda(\xi) \rightarrow 1$, which is achieved by letting to point ξ go to infinity. In a similar manner we find that the gauge invariance of the fields $\mathcal{A}_0(x,\xi)$ and $\mathcal{A}_i(x,\xi)$ is restored when $\Lambda(\xi) \rightarrow 1$. As a consequence of this one can construct Yang-Mills gauge

invariant, not merely covariant, fields. On the other hand, if we had selected from the beginning the path in the form of a spacelike straight line, where the reference point ξ is in spatial infinity, we would arrive at an expression for $\mathcal{A}_\mu$ that coincides with the axial gauge. In order to show this more clearly it is instructive to repeat the above derivation for the infinite reference point case.

B. The Axial gauge

First let us discuss the Abelian case. For this purpose we start by considering the gauge invariant field given by

$$\mathcal{A}_\mu(x) = A_\mu(x) + \partial_\mu \left(-\int_{-\infty}^{0} A_\sigma(z) \frac{\partial z^\sigma}{\partial \xi} d\xi \right). \tag{22}$$

As already expressed the reference point ξ is in infinity and $C_{\xi x}$ is a spacelike path or contour that to join the points ξ and x. Now, $z^\mu(x,\xi)$ are four arbitrary single-valued differential functions that satisfy

$$z^\mu(x,0) = x^\mu, \tag{23}$$

$$z^\mu(x, \xi \to -\infty) = \infty(spatial). \tag{24}$$

Just as for the Poincaré gauge case, after a brief calculation, from (22) one gets

$$\mathcal{A}_\mu(x) = \int_{-\infty}^{x} F_{\nu\sigma}(z) \frac{\partial z^\sigma}{\partial x^\mu} \frac{\partial z^\nu}{\partial \xi} d\xi. \tag{25}$$

As before, the potentials $\mathcal{A}_\mu$ have the property that they can be expressed in terms of the field strength $F_{\mu\nu}$. To illustrate a practical use of expression (25), we can write $z^\mu(x,\xi) = x^\mu + \xi n^\mu$ with the vector $n^\mu = (0,0,0,1)$ and $-\infty < \xi \leq 0$, in which case $\mathcal{A}_\mu(x)$ is said to be in the axial gauge.

The next step is to extend this analysis to the non-Abelian case. Basically, we follow the same procedure as we developed in the Poincaré gauge case. According to this idea one writes

$$A_\mu(x,\xi) = A_\mu(x) + \partial_\mu\left(-\int_{-\infty}^{0} A_\sigma(z)\frac{\partial z^\sigma}{\partial\xi}\right) - ig\int_{-\infty}^{0} d\xi \frac{\partial z^\sigma}{\partial\xi}\frac{\partial z^\rho}{\partial x^\mu}\left[A_\sigma(z), A_\rho(z)\right], \tag{26}$$

which by its turn is rewritten as

$$A_\mu(x,\xi) = -\int_{-\infty}^{0} d\xi \frac{\partial A_\sigma}{\partial z^\rho}\frac{\partial z^\rho}{\partial x^\mu}\frac{\partial z^\sigma}{\partial\xi} + \int_{-\infty}^{0} d\xi \frac{\partial A_\sigma}{\partial\xi}\frac{\partial z^\sigma}{\partial x^\mu} - ig\int_{-\infty}^{0} d\xi \frac{\partial z^\sigma}{\partial\xi}\frac{\partial z^\rho}{\partial x^\mu}\left[A_\sigma(z), A_\rho(z)\right]. \tag{27}$$

By means the definition for the field strength in terms of the vector potential

$$G_{\nu\rho} = \partial_\sigma A_\rho - \partial_\rho A_\sigma - ig\left[A_\sigma, A_\rho\right], \tag{28}$$

the expression (27) then becomes

$$A_\mu(x,\xi) = \int_{-\infty}^{0} dz^\nu G_{\nu\sigma}(z(\xi))\frac{\partial z^\sigma}{\partial x^\mu}. \tag{29}$$

In other words, one obtains a non-Abelian inversion formula which uniquely expresses the potentials $A_\mu(x,\xi)$ in terms of the gauge field strengths $G_{\mu\nu}$, like in the Poincaré gauge case. We now proceed to perform the gauge transformation (12) and (13) with the operator (14), and now with the spacelike path of integration running from infinity to x. Proceeding in analogy to the Poincaré gauge case and after some manipulations we find that $\mathcal{G}_{\mu\nu}(x,\xi) \equiv G^{\Lambda}_{\mu\nu}(x,\xi)$ and $\mathcal{A}_\mu(x,\xi) \equiv A^{\Lambda}_\mu(x,\xi)$ are gauge invariant.

III. INTERQUARK ENERGY

The goal of this section is to implement the above general considerations in a concrete application. So we proceed to calculate the interaction energy between external probe sources in a non-Abelian pure Yang-Mills theory. This calculation will be carry out with the help of the previously discussed string variables. Before considering explicitly the interquark energy, we shall first reexamine the canonical quantization for the Yang-Mills field coupled to an external source J^0 from the viewpoint of hamiltonian dynamics. We start from the Lagrangian density

$$\mathcal{L} = -\frac{1}{4}Tr\left(F_{\mu\nu}F^{\mu\nu}\right) - A_0^a J^0 = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - A_0^a J^0. \tag{30}$$

Here $A_\mu(x) = A^a_\mu(x) T^a$, where T^a is a hermitian representation of the semi-simple and compact gauge group; and $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$, with f^{abc} the structure constants of the gauge group. The Dirac procedure [13] as applied to (30) is straightforward. The canonical momenta are $\Pi^{a\mu} = -F^{a0\mu}$, which results in the usual primary constraint $\Pi^a_0 = 0$, and $\Pi^{ai} = F^{ai0}$. This allows us to write the following canonical Hamiltonian:

$$H_c = \int d^3x \left(-\frac{1}{2}\Pi^a_i \Pi^i_a + \Pi^a_i \partial^i A^0_a + \frac{1}{4} F^a_{ij} F^{aij} - g f_{abc} \Pi^a_i A^0_b A^i_c + A^a_0 J^0 \right). \tag{31}$$

The secondary constraint generated by the time preservation of the primary constraint is now

$$\Omega^{(1)}_a(x) = \partial_i \Pi^i_a + g f_{abc} A^i_b \Pi^c_i - J^0 \approx 0. \tag{32}$$

It is easy to check that there are no more constraints in the theory, and that both constraints are first class. The corresponding total (first class) Hamiltonian that generates the time evolution of the dynamical variables is given by

$$H = H_c + \int d^3x \left(c_0(x) + c_1(x)\, \Omega^{(1)}_a(x) \right), \tag{33}$$

where c_0 and c_1 are arbitrary functions. Since $\Pi^a_0 \approx 0$ for all time and $\dot{A}^a_0(x) = [A^a_0(x), H] = c_0(x)$, which is completely arbitrary, we discard $A^a_0(x)$ and $\Pi^0_a(x)$ since they add nothing to the description of the system. The Hamiltonian then takes the form

$$H = \int d^3x \left(-\frac{1}{2}\Pi^a_i \Pi^i_a + \frac{1}{4} F^a_{ij} F^{aij} + c^a(x) \left(\partial^i \Pi^a_i + g f_{abc} A^i_b \Pi^c_i - J^0 \right) \right), \tag{34}$$

where $c^a(x) = c_1(x) - A^a_0(x)$.

Therefore we have one first class constraint $\Omega^{(1)}_a(x)$, which appears at the secondary level. In order to break the gauge freedom of the theory, it is necessary to impose one constraint such that the full set of constraints becomes second class. From the previous section, we choose

$$\Omega^{(2)}_a(x) = \int_0^1 d\lambda\, (x-\xi)^k A^{(a)}_k(\xi + \lambda(x-\xi)) \approx 0. \tag{35}$$

There is no essential loss of generality if we restrict our considerations to $\xi^k = 0$; accordingly, (35) becomes

$$\Omega_a^{(2)}(x) = \int_0^1 d\lambda x^k A_k^a(\lambda x) \approx 0. \tag{36}$$

Standard techniques for constrained systems then lead to the following Dirac brackets

$$\left\{A_i^a(x), A_b^j(y)\right\}^* = 0 = \left\{\Pi_i^a(x), \Pi_b^j(y)\right\}^*, \tag{37}$$

$$\left\{A_i^a(x), \Pi^{bj}(y)\right\}^* = \delta^{ab}\delta_i^j\delta^{(3)}(x-y) - \int_0^1 d\lambda \left(\delta^{ab}\frac{\partial}{\partial x^i} - gf^{abc}A_i^c(x)\right) x^j \delta^{(3)}(\lambda x - y). \tag{38}$$

Also we mention that equivalent Dirac brackets were calculated independently in reference [14]. This completes our brief review of the canonical quantization for the Yang-Mills field.

Now we move on to compute the energy of the external field of static charges where a fermion is localized at $\mathbf{y}'$ and an antifermion at $\mathbf{y}$. In order to accomplish this purpose we will calculate the expectation value of the energy operator H in the physical state $|\Omega\rangle$, which will denote by $\langle H\rangle_\Omega$. Using Eq.(34) we see that $\langle H\rangle_\Omega$ reads

$$\langle H\rangle_\Omega = tr\,\langle\Omega| \int d^3x \left(-\frac{1}{2}\Pi_i^a\Pi^{ia} + \frac{1}{4}F_{ij}^a F^{aij}\right)|\Omega\rangle . \tag{39}$$

Since the fermions are taken to be infinitely massive (static), this can be further simplified as

$$\langle H\rangle_\Omega = tr\,\langle\Omega| \int d^3x \left(-\frac{1}{2}\Pi_i^a(x)\Pi^{ia}(x)\right)|\Omega\rangle . \tag{40}$$

It is now important to notice that, as was first established by Dirac [15], the physical states $|\Omega\rangle$ correspond to the gauge invariant ones. It is worth recalling at this stage that in the Abelian case $|\Omega\rangle$ may be written as [7]

$$|\Omega\rangle = \overline{\psi}(\mathbf{y}) \exp\left(ie\int_{\mathbf{y}'}^{\mathbf{y}} dz^i A_i(z)\right)\psi(\mathbf{y}')|0\rangle , \tag{41}$$

where $|0\rangle$ is the physical vacuum state and the line integral appearing in the above expression is along a spacelike path from the point at $\mathbf{y}'$ to $\mathbf{y}$, on a fixed time slice. As a result of this

the fermion fields are now dressed by a cloud of gauge fields. As before, the above expression must be extended since we are dealing with non-Abelian fields. Thus, on the basis of the discussion in the previous section, we can write a state which has a fermion at $\mathbf{y}'$ and an antifermion at $\mathbf{y}$ as

$$\left|\Omega\right\rangle = \overline{\psi}\left(\mathbf{y}\right) P \exp\left(ig \int_{\mathbf{y}'}^{\mathbf{y}} dz^i A_i\left(z\right)\right) \psi\left(\mathbf{y}'\right)\left|0\right\rangle . \tag{42}$$

As before, the line integral is along a spacelike path on a fixed time slice, and $\left|0\right\rangle$ is the physical vacuum state.

The above analysis give us an opportunity to compare our work with the standard Wilson loop procedure [16], to make sure that the known results are recovered from the general expression (42) in the weak coupling limit. In effect, due to asymptotic freedom, the short distance behavior of the interquark potential is determined by perturbation theory. According to this, at weak coupling, one can expand

$$P \exp\left(ig \int_{\mathbf{y}'}^{\mathbf{y}} dz^i A_i\left(z\right)\right) = P\left(1 + ig \int_{\mathbf{y}'}^{\mathbf{y}} dz^i A_i^a\left(z\right) T^a + ...\right). \tag{43}$$

This implies that, at lowest order in g, the non-Abelian generalization of the dressing framework is the same as in the Abelian theory. In other terms, this means that at short distances we get the Coulomb potential with g^2 multiplied by a group factor, as we will now show.

It is appropriate to start first by reconsidering how the gauge invariant state (dressed) represent charged particles with a static field electric on a line or, more precisely, on a tube. Let $\left|E\right\rangle$ be an eigenvector of the electric field operator $E_i^a(x)$, with eigenvalue $\varepsilon_i^a(x)$:

$$E_i^a\left(x\right)\left|E\right\rangle = \varepsilon_i^a\left(x\right)\left|E\right\rangle . \tag{44}$$

We then focus our attention towards examining the state

$$U\left(\mathbf{y}', \mathbf{y}\right)\left|E\right\rangle \equiv \overline{\psi}\left(\mathbf{y}\right)\left(1 + ig \int_{\mathbf{y}'}^{\mathbf{y}} dz^i A_i^c\left(z\right) T^c\right) \psi\left(\mathbf{y}'\right)\left|E\right\rangle . \tag{45}$$

Moreover, it follows from (38) and (44) that

$$E_i^a(x)\,U(\mathbf{y}',\mathbf{y})\,|E\rangle = \left(\varepsilon_i^a(x) + g\int_{\mathbf{y}'}^{\mathbf{y}} dz_i\delta^{(3)}(x-z)\,T^a\right)U(\mathbf{y}',\mathbf{y})\,|E\rangle\,. \tag{46}$$

This means that $U(\mathbf{y}',\mathbf{y})\,|E\rangle$ is another eigenvector of $E_i^a(x)$ with eigenvalue of $\varepsilon_i^a(x) + g\int_{\mathbf{y}'}^{\mathbf{y}} dz_i\delta^{(3)}(x-z)\,T^a$. As in [7], by employing (40)and (46) we then evaluate the energy in the presence of the static charges. Hence we once again obtain

$$\langle H\rangle_\Omega = \langle H\rangle_0 + \frac{1}{2}g^2 trT^aT^a\int_{\mathbf{y}'}^{\mathbf{y}} dz^i\int_{\mathbf{y}'}^{\mathbf{y}} dz_i'\delta^{(3)}(z-z')\,, \tag{47}$$

where $\langle H\rangle_0 = \langle 0|H|0\rangle$. Recalling that the integrals over z^i and z_i' are zero except on the contour of integration, we obtain the following interaction energy

$$V = \frac{1}{2}g^2k(trT^aT^a)|\mathbf{y}-\mathbf{y}'|, \tag{48}$$

where $k = \delta^{(2)}(0)$. Writing the purely group theoretic factor $trT^aT^a = C_2(F)$, (48) can be further expressed as

$$V = \frac{1}{2}g^2C_2(F)k|\mathbf{y}-\mathbf{y}'|. \tag{49}$$

This expression may look peculiar, but it is nothing but the familiar Coulomb energy as was discussed in [7]. Here, however, we call the attention to the fact that, as in the Abelian case, the term $\frac{g^2}{2}\int d^3x\left(\int_{\mathbf{y}'}^{\mathbf{y}} dz_i\delta^{(3)}(x-z)\right)^2$ reproduces exactly the Coulomb energy. In this way one obtains the standard result for the interquark potential

$$V = -\frac{1}{4\pi}g^2C_2(F)\frac{1}{|\mathbf{y}-\mathbf{y}'|}. \tag{50}$$

We further note that, as was explained in [7], a modified form for the state (42) in the Poincaré gauge is equivalent to the Coulomb gauge, which, too, leads to the expression (50).

As a final comment, notice that the results of this section hinge on the constraints structure of the theory we have discussed. Thus it seems straightforward to extend to the g^4 order the calculation that we have developed. We expect to report on progress along these lines soon.

IV. ACKNOWLEDGMENTS

The author would like to thank I. Schmidt for reading the manuscript and for his support. Work supported in part by Fondecyt (Chile) grant 1000710, and by a Cátedra Presidencial en Ciencias (Chile).

REFERENCES

[1] B. Hatfield, *Quantum Field Theory of Point Particles and Strings* (Addison Wesley, Redwood City, 1992).

[2] A. Sisakyan, O. Shevchenko and I. Solovtsov, Sov. J. Part. Nucl. **21**, 285 (1990).

[3] S. Caracciolo, G. Curcci and P. Menotti, Phys. Lett. **B113**, 311 (1982).

[4] M. Lavelle and D. McMullan, Phys. Rep. **279**, 1 (1997).

[5] L. Prokhorov and S. Shabanov, Int. J. Mod. Phys. **A7**, 7815 (1992)

[6] P. Gaete, Z. Phys. **C76**, 355 (1997).

[7] P. Gaete, Phys. Rev. **D59**, 127702 (1999).

[8] P. Gaete and I. Schmidt, hep-th/9908123.

[9] K. Huang, *Quarks Leptons and Gauge Fields* (World Scientific, Singapore, 1982).

[10] S. Mandelstam, Phys. Rev. **175**, 1580 (1968).

[11] P. Gaete, C. A. P. Galvão and B. Pimentel, Int. J. Mod. Phys. **A6**, 5373 (1991).

[12] C. A. P. Galvão and B. Pimentel, C.B.P.F. preprint CBPF-NF-006/88.

[13] P. A. M. Dirac, *Lectures on Quantum Mechanics* (Yeshiva University, New York, 1964).

[14] L. Lukaszuk, E. Leader and A. Johansen, hep-th/9906026

[15] P. A. M. Dirac, Can. J. Phys. **33**, 650 (1955).

[16] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, Reading, Massachusetts, 1995).